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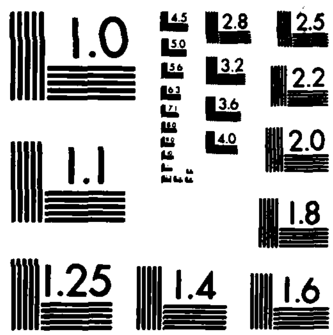

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ALGORITHMS AND HEURISTICS FOR VARIABLE-YIELD LOT-SIZING

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Joseph B. Mazzola
William F. McCoy*
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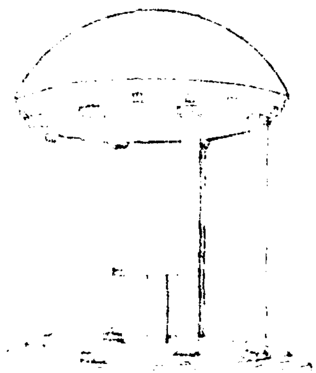

SCHOOL OF BUSINESS ADMINISTRATION
AND
CURRICULUM IN
OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

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ALGORITHMS AND HEURISTICS FOR VARIABLE-YIELD LOT-SIZING

Technical Report #24

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Decision Control Models in Operations Research

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We consider the multiperiod lot-sizing problem in which the production yield (the proportion of usable goods) is variable according to a known probability distribution. A dynamic programming algorithm for an arbitrary sequence of demand requirements is presented. We review two economic order quantity (EOQ) models for the stationary demand continuous-time problem and derive an EOQ model when the production yield follows a binomial distribution and backlogging of demand is permitted. Heuristics based on the EOQ model are

discussed, and a computational evaluation of these heuristics is presented. The heuristics consistently produced near optimal lot-sizing policies for problems with stationary and cyclic demands.

Foreward

As part of the on-going program in "Decision Control Models in Operations Research," Professor Joseph Mazzola, Mr. William McCoy and Professor Harvey Wagner have examined economic order quantities when the amount supplied is random. This situation occurs in many industrial settings, such as the manufacturing of electronics. The report provides easy-to-compute formulas that give close to optimal quantities. The formulas are straightforward modifications to the classic economic lot-size model. The approximations are tested with differing economic and deterministic demand parameter settings. Other related reports dealing with this program are given on the following pages.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-Principal Investigator

MacCormick, A. (1974), *Statistical Problems in Inventory Control*, ONR and ARO Technical Report 2, December 1974, School of Organization and Management, Yale University, 244 pp.

Estey, A. S. and R. L. Kaufman (1975), *Multi-Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 9)*, ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

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Blazer, D. (1983), Testing the Cost Effectiveness of an Inventory Filtering Rule Using Empirical Data, Technical Report #23, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 24pp.

Mazzola, J.B., W.F. McCoy and H.M. Wagner (1983), Algorithms and Heuristics for Variable-Yield Lot-Sizing, Technical Report #24. School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 18pp.

Blazer, D. (1983), Implementation Strategy for an Inventory Filtering Rule, Technical Report #25, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 20pp.

Abstract

We consider the multiperiod lot-sizing problem in which the production yield (the proportion of usable goods) is variable according to a known probability distribution. A dynamic programming algorithm for an arbitrary sequence of demand requirements is presented. We review two economic order quantity (EOQ) models for the stationary demand continuous-time problem and derive an EOQ model when the production yield follows a binomial distribution and backlogging of demand is permitted. Heuristics based on the EOQ model are discussed, and a computational evaluation of these heuristics is presented. The heuristics consistently produced near optimal lot-sizing policies for problems with stationary and cyclic demands.

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Algorithms and Heuristics for Variable-Yield Lot-Sizing

by

Joseph B. Mazzola, William F. McCoy, and Harvey M. Wagner

1. Introduction

The general multiperiod lot-sizing problem involves trading off of setup, production, inventory carrying, and stockout costs over a planning horizon. The solution is a policy that minimizes the sum of these various costs while satisfying demand over the planning horizon.

The earliest solution to the lot-sizing problem was formulated by Harris in the early 1900s; this model is commonly called the Wilson Economic Order Quantity (EOQ) model. In 1958, Wagner and Whitin [17] introduced a dynamic lot-sizing algorithm for solving the multiperiod problem with varying deterministic demands. Under certain conditions, the use of (s,S) models was shown to be optimal by Beckmann [2]. For a review of the early literature on the basic lot-sizing model, the reader is directed to Veinott [16].

In this paper, we address a generalization in which the production yield (the proportion of nondefective goods obtained from a production run) is a random variable and specified by a probability distribution. The variable yield lot-sizing problem is important to high technology industries in which many of the manufacturing processes experience less than perfect yields. This phenomenon could possibly be attributed to extremely rigid tolerance specifications or perhaps to various complexities in the production processes themselves. A particular example arises in the silicon chip industry. Due to the inherent nature of the manufacturing process, many firms in the industry

experience considerable fluctuation in their production yields. Consequently, these firms often suffer excessive inventory carrying costs in an attempt to meet product demands. The standard lot-sizing models do not adequately address this problem.

One of the first discussions of the variable-yield lot-sizing problem appears in Karlin [7, 8]; several results for the single- and multiperiod versions of the problem are presented for the case when no setup costs are incurred. Silver [14] develops a general EOQ model for the variable-yield lot-sizing problem. He shows that the only additional factors affecting the economic order quantity are the mean and standard deviation of the yield distribution.

Recently, Shih [13] also addresses the variable-yield lot-sizing problem. The first of his models is an EOQ model; the second is a single-period model in which there is a probabilistic percentage of defective items in the lot as well as probabilistic demand. In both of these models, Shih assumes that the probability distribution of the yield is independent of the order quantity. Kelly [9] examines a variable-yield batch sizing problem involving a single one-time demand.

We present an algorithm and two heuristics for the multiperiod variable-yield lot-sizing problem. In the next section, we provide a dynamic programming formulation. In section three, we consider the continuous-time version of the problem. We extend Silver's model [14] to the case in which backlogging of demand is permitted. We give specific results for the situation in which the variable yield obeys a binomial distribution.

In section four, we define two easy-to-implement heuristics for the multiperiod variable yield lot-sizing problem. In the last section, we report the results of the computational testing of these heuristics on two classes of

problems with a binomial yield distribution. These heuristic procedures are quite good in that they consistently provide policies with expected costs that are within 0.4% of the optimal cost as determined by the dynamic programming algorithm.

2. Dynamic Programming Algorithm

Dynamic programming algorithms for deterministic as well as stochastic lot-sizing models are well precedented in the literature (for example, see [3], [4], [5], [6], [10], and [17]).

Assume that demand over the next N time periods comprising a planning horizon is deterministic and given by $D_1, D_2, \ldots, D_N$. Given the inventory level at the beginning of any period t, we decide the production size $Q_t \geq 0$ (in units).

Assume that production capacity is $M > 0$ units per period. A setup cost of K dollars is incurred each time $Q_t > 0$. The variable production cost is c_v dollars/unit, and since the lead time is less than one period, all (nondefective) units produced during a period can be used to meet that period's demand, which occurs at the end of the period.

For a lot of size Q units, the (integer) number x of usable (nondefective) units follows a discrete probability distribution $\varphi(x|Q)$. Assume all defective units are discarded with no salvage value. The nondefective units are used to meet either current (or past) demand or placed in finished goods inventory.

A carrying cost of c_h dollars/unit is charged each period for ending inventory. Stocking out of demand is permitted, and all stockouts are back-ordered. A stockout cost of c_s dollars/unit is assessed each period for any demand that has not been filled.

Define $f_n^*(i)$ as the expected total cost of an optimal lot-sizing policy with an initial inventory level of i units and with n periods remaining in the planning horizon. Assuming that $f_{n-1}^*(i)$ has been determined for all possible beginning inventory levels i, then for any feasible production quantity Q, let

$$f_n(Q|i) \equiv \delta(Q)K + c_vQ + \sum_{x=0}^{Q} [c_h(i+x-D_t)^+ + c_s(D_t-x-i)^+ + f_{n-1}^*(i+x-D_t)]\varphi(x|Q), \tag{1}$$

where

$$\delta(y) = \begin{cases} 1 & \text{if } y > 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$(y)^+ = \max\,\{y,0\}\ ,$$

and

$$t \equiv N-n+1\ .$$

We then define

$$f_n^*(i) = \min_{0 \le Q \le M} \{f_n(Q|i)\}, \qquad -B \le i \le S, \tag{2}$$

where $B > 0$ is a bound on the total number of backlogged units and S is storage capacity. Also, let $\mathcal{Q}_n^*(i) = \{Q|f_n(Q|i) = f_n^*(i)\}$ be the set of all optimal production quantities. The model requires that the ending conditions $f_0^*(i)$ be specified for all possible end-of-horizon inventory levels i.

3. <u>EOQ Models</u>

We now consider the continuous time variable yield lot-sizing problem. Assuming that demand is constant and deterministic, lead time is known, and orders are produced instantaneously, several authors have developed economic order quantity (EOQ) models for the problem.

In [13], Shih extends the basic EOQ model without backlogging of demand to include the presence of a variable yield. In this case, the fraction ξ of usable (nondefective) goods is stochastic with a (known) probability density of $\Phi(\xi)$; thus the yield distribution is independent of the production quantity Q. For the case of no stockouts, the total annual cost function may be written as

$$\frac{K + c_v Q + (c_H/2D)(E(\xi^2)Q^2)}{E(\xi)Q/D}, \tag{3}$$

where K is the setup cost, c_v is the variable production cost (per unit), c_H is the annual holding cost per unit ($c_H = 12c_h$ if each period equals one month), D is the annual rate of demand, $E(u)$ is the expected value of u under the distribution $\Phi(\xi)$, and the decision variable Q is the quantity scheduled for production. The resulting optimal production quantity is

$$Q^* = \frac{1}{\sqrt{E(\xi^2)}} \quad \sqrt{2KD/c_H} \ . \tag{4}$$

Note the scaling factor $1/\sqrt{E(\xi^2)}$ appearing in (4). One might expect the scaling factor to be $1/E(\xi)$; however, the appearance of the second moment in (4) comes from the calculation of the holding cost. Shih's model [13] can be extended to allow for stocking out and backlogging of demand.

We assume that the actual yield <u>is</u> a function of the number of goods Q produced; that is, the variable yield distribution is specified as $\varphi(x|Q)$, where x is the number of usable goods resulting from a lot of size Q. Silver [14] presents an economic order quantity model for this case. Specifically, Silver shows that the expected total annual cost (exclusive of the expected annual variable production costs) is given by

$$\frac{K + (c_H/2D)(E(x^2|Q))}{E(x|Q)/D}, \tag{5}$$

where $E(u|Q)$ denotes the expected value of u given Q under the distribution $\varphi(x|Q)$.

It is shown that when $E(x|Q) = bQ$, for some constant b, and the standard deviation $\sigma_{x|Q} = \sigma$, then the economic order quantity is

$$Q^* = \frac{1}{b}\sqrt{(2KD/c_H) + \sigma^2} . \tag{6}$$

Alternatively, if $\sigma_{x|Q} = kQ$, where k is a constant, then

$$Q^* = \sqrt{2KD/[c_H(b^2+k^2)]} . \tag{7}$$

This model is now extended to the case in which backlogging of demand is permitted and all backlogged demand must be met. Assume a lead time of 0. As in the basic EOQ model in which backlogging of demand is permitted, we also assume that production is not scheduled until the (backlog) inventory level drops to $i < 0$. Once this reorder level is reached, an amount Q is produced resulting in an amount $x \leq Q$ of usable goods that arrive, where the probability of realizing x is given by the discrete distribution $\varphi(x|Q)$. Any backlog that exists is satisfied and $x + i$ is held in inventory. This inventory is depleted at a known rate D, and the (average) amount of inventory held is $(x + i)^2/2D$. Since production is not scheduled until a backlog of $|i|$ units has accumulated, the average number of backlogged units is $i^2/2D$. At the end of this cycle, a lot of size Q is scheduled for production and the process repeats.

The expected holding cost is given by

$$\sum_{x=0}^{Q} [c_H(x+i)^2/2D]\varphi(x|Q)] = (c_H/2D)[E(x^2|Q) + 2iE(x|Q) + i^2].$$

Therefore, given i, the expected total cost for a single replenishment cycle is

$$K + c_v Q + (c_H/2D)[E(x^2|Q) + 2iE(x|Q) + i^2] + c_S i^2/2D. \tag{8}$$

In order to transform the costs to expected average cost per year, expression (8) must be scaled by the appropriate time interval. It is no longer appropriate, however, to scale the expression by x/D since each random outcome x induces a different duration between orders. Assuming that the time between production runs is independently and identically distributed, since the model requires that we schedule production from the same level of inventory each time, the production cycle can be viewed as a renewal process.

To find the cost per unit time, we have from renewal theory [11]

$$\frac{\text{Total cost accumulated by time } t}{t} \to \frac{E(\text{single-cycle cost})}{E(\text{time between production runs})}$$

as $t \to \infty$. Thus, dividing (8) by the expected time between production runs $E(x|Q)/D$, the following expression for expected total annual cost is obtained

$$\frac{K + c_v Q + (c_H/2D)[E(x^2|Q) + 2iE(x|Q) + i^2] + c_S i^2/2D}{E(x|Q)/D}. \tag{9}$$

If we now assume that each item produced has a probability p of being usable and that this occurrence is independent of other units produced, then a production lot of size Q can be viewed as a sequence of Q Bernoulli trials. Hence, the probability of obtaining x usable goods (a yield of x/Q) in a lot of size Q is given by the binomial probability of obtaining x successes in Q trials

$$\varphi(x|Q) = \binom{Q}{x} p^x (1-p)^{Q-x} .$$

For this distribution $E(x|Q) = pQ$ and $E(x^2|Q) = pQ(1+p(Q-1))$.

Using the binomial distribution in expression (9), we have that expected annual cost is

$$\frac{K + c_v Q + (c_H/2D)[(pQ(1+p(Q-1)) - 2ipQ + i^2] + c_S i^2/2D}{pQ/D} . \tag{10}$$

Thus, the optimal Q^* and i^* are

$$Q^* = (1/p)\sqrt{2KD/c_H}\ \sqrt{(c_H + c_S)/c_S} . \tag{11}$$

$$i^* = \frac{-1}{\sqrt{c_S(c_H + c_S)}}\ \sqrt{2KDc_H} . \tag{12}$$

4. Heuristic Procedures

Basic EOQ models require stringent assumptions; however, these models are readily adaptable to more general situations by modifying them into heuristic decision rules. We now present two heuristic rules for the multiperiod variable yield lot-sizing problem.

The general lot-sizing problem involves dynamic demand, and thus it is common practice to use the sum of the future twelve monthly demands D_t as the value for annual demand D. Each of the heuristic procedures presented below uses this value in the computation of the economic order quantity Q^* and the reorder point i^* as determined by (11) and (12).

The basic EOQ model assumes that demand occurs continuously. The multi-period, variable yield lot-sizing problem, on the other hand, is a discrete time problem. Consequently, if one attempts to employ a straightforward application of the well-known (s,Q) decision rule, with $s = i^*$ and $Q = Q^*$, situations often arise in which the inventory level at the beginning of one period is above the reorder point s, but by the beginning of the next period the inventory level will have fallen well below the reorder point. Thus, in some sense, such a decision rule may result in waiting "too long" to produce. The occurrence of the phenomenon suggests a modification of the reorder point i^*.

Since the EOQ reorder level i^* already optimally balances holding and stockout costs, it appears reasonable to adopt a new production policy that calls for making a production run whenever the current inventory level i is such that (after experiencing this period's demand) the resulting inventory level at the beginning of the next period will fall strictly below i^* (if no run is made). Thus, the following dynamic or hybrid (s,Q) decision rule is suggested. If in any period t, the beginning inventory level is less than or equal to $s_t = D_t + i^* - 1$, produce Q^* units; otherwise, do not produce. This rule may be looked upon as an (s_t,Q) decision rule, where the reorder point is a function of the period t; the HEUR 1 heuristic utilizes this rule.

The other heuristic is based on the notion that an (s,S) model (sometimes called a min max model) may be more appropriate for this particular problem. The intuition behind adopting such a model stems largely from the wide success of (s,S) models for many other types of stochastic inventory

models. (See [1], [15], and [20].) Based on empirical observation, we heuristically choose to set $S = Q^*$ where Q^* is the economic order quantity used in the earlier heuristic. Thus, the second heuristic HEUR 2 may be looked upon as an (s_t,S) decision rule in which an amount Q^*-i is produced whenever the inventory level i at the beginning of any period t is less than or equal to D_t+i^*-1.

The two heuristic rules are summarized in Table 1.

Table 1. Summary of the Heuristic Procedures

Heuristic Procedure (k)	Name	Type Policy	Order Quantity† (Q_k)	Reorder Point (s_k)
1	HEUR 1	(s_t,Q)	Q^*	D_t+i^*-1
2	HEUR 2	(s_t,S)	Q^*-i	D_t+i^*-1

†i is the beginning period inventory level.

The recursive structure of the dynamic programming algorithm may be used to calculate the expected cost of the heuristic procedures. Recall from Section 2 that $f_n^*(i)$ is the expected cost of an optimal lot-sizing policy given an initial inventory level of i and n periods left to go. Similarly, define $f_n^k(i)$ to be the expected cost of a lot-sizing policy given by decision rule k, where $k = 1,2$ corresponds to the two heuristics, given an initial inventory level of i and n periods left to go in the planning horizon. Thus

$$f_n^k(i) = \begin{cases} \delta(Q_k)K + c_v Q_k + \sum_{x=0}^{Q_k} [c_h(i+x-D_t)^+ + c_s(D_t-x-i)^+ \\ \qquad\qquad + f_{n-1}^k(i+x-D_t)]\varphi(x|Q_k) & \text{if } i \le s_k, \\ c_h(i-D_t)^+ + c_s(D_t-i)^+ + f_{n-1}^k(i-D_t) & \text{otherwise,} \end{cases}$$

where $t \equiv N-n+1$ and Q_k and s_k are the order quantity and reorder point of the k^{th} decision rule (as given in Table 1). In this manner, we calculate $f_n^k(i)$ by backward recursion and thus calculate the expected cost of using policy k over the planning horizon for the desired beginning of the horizon inventory level.

5. Computational Results

The EOQ heuristics as well as the dynamic programming algorithm were tested and compared on two groups of problems in which the mean demand rates were 10 and 25 units/period, respectively. Two distinct demand patterns were represented within each group. One pattern has constant demand per period, while the other has sinusoidal demand. These demand patterns are given in Table 2.

The production yield is assumed to follow a binomial distribution. We considered the three cases in which the probability p of an individual unit being nondefective is 80%, 50%, and 20%. In this manner, we can compare the relative behavior of the heuristics in the face of different production yields.

Three different cost structures were considered for each demand pattern and each production yield. The various costs were set relative to one another so that the expected number of production runs, as determined by the binomial

yield EOQ model, is 2, 4, or 6 times per year. The unit holding cost c_h is set at $1. Incorporating the well known "Newsboy heuristic rule" [6], which suggests that $c_h/(c_h+c_s)$ = probability of stockout, the unit stockout cost c_s is set equal to $19 (thus approximating a 95% service level). A variable production cost $c_v = \$4.00$ is assumed. The setup costs k are set so that the EOQ formula (11) yields the desired number of expected production runs per year, and they are given in Tables 3 and 4.

Table 2. Annual Demand Patterns

Average Monthly Demand	Month Pattern	1	2	3	4	5	6	7	8	9	10	11	12
$\bar{D} = 10$	Constant	10	10	10	10	10	10	10	10	10	10	10	10
	Sinusoidal	10	13	14	15	14	13	10	7	6	5	6	7
$\bar{D} = 25$	Constant	25	25	25	25	25	25	25	25	25	25	25	25
	Sinusoidal	25	31	36	38	36	31	25	19	14	12	14	19

For each problem, the maximum production amount M is set equal to min $\{D/p, 600\}$, where D is the total demand for one year, p is the probability of an individual unit being nondefective, and the 600 unit maximum arises from a computer storage capacity limitation. The maximum inventory storage capacity S is set equal to the corresponding value of M in each case. The maximum number of units that could be backlogged at any point in time is set equal to the total demand D for one year.

In order to compare the relative performance of the various procedures over a twelve month planning horizon, it is necessary to establish appropriate ending conditions. This is accomplished by first solving each problem to optimality over an initial 24 month time period using the dynamic programming algorithm with

$$f_0(i) = \begin{cases} c_h i & \text{if} \quad i \geq 0 , \\ - c_s i & \text{otherwise} . \end{cases}$$

For each problem, the resulting cost vector $f^*_{24}(i)$ is then "normalized" by subtracting the smallest entry in the vector from each of its components. In this way, the resulting vector contains the relative (nonnegative) penalties of starting out in each possible inventory position when 24 months remain in the horizon. This modified vector is then used as an <u>end</u>-<u>of</u>-<u>horizon</u> cost vector for the twelve-month costs comparisons discussed below.

Tables 3 and 4 report the twelve-month cost of each of the lot-sizing procedures for an initial inventory level of 0 units. Using these tables we also can assess directly the relative performance of the heuristic procedures by comparing the degree (in terms of percent) to which they approach the optimal expected cost (as given by the dynamic programming solution). In order to give some indication of the computational effort required to solve these problems to optimality, we mention that using an IBM 3081 computer with a FORTRAN H level compiler, on average the solution of a problem with a mean demand rate of 10 units per month required 10.2 minutes of CPU time and the solution of a problem with a mean demand rate of 25 units per month required 46.5 minutes of CPU time.

We immediately observe that both heuristics consistently provide lot-sizing policies with expected costs that are within 0.4% of optimality. The heuristic rule given by HEUR 2 almost always dominates that of HEUR 1; in addition, with one exception, HEUR 2 always provides as good or better solutions to the problems with constant demand. The exception occurred in the case when the economic order quantity was truncated to 600 because of a computer storage capacity constraint. Note, however, that even in this case, both heuristics continue to provide excellent quality solutions.

Table 3. Expected 12 Month Cost of Lot-Sizing Policies (and Proximity of Optimality of Heuristics) with a Mean Demand Rate of 10 Units per Month

Demand	Expected No. of Setups	K	EOQ†	j*†	DP	HEUR 1	HEUR 2
80% Probability of Individual Unit being Nondefective							
Constant	2	171.00	75	-3	16682.1	16685.6 (0.02%)	16682.9 (0.00%)
	4	42.75	38	-1	15889.5	15899.9 (0.07%)	15890.7 (0.01%)
	6	19.00	25	-1	15619.7	15638.1 (0.12%)	15627.6 (0.05%)
Sinusoidal	2	171.00	75	-3	16711.3	16725.7 (0.09%)	16728.5 (0.10%)
	4	42.75	38	-1	15973.6	15999.8 (0.16%)	15995.8 (0.14%)
	6	19.00	25	-1	15718.4	15744.4 (0.17%)	15761.9 (0.28%)
50% Probability of Individual Unit being Nondefective							
Constant	2	171.00	120	-3	32477.7	32480.2 (0.01%)	32479.8 (0.01%)
	4	42.75	60	-1	31537.5	31541.1 (0.01%)	31539.2 (0.01%)
	6	19.00	40	-1	31218.2	31227.0 (0.03%)	31222.8 (0.01%)
Sinusoidal	2	171.00	120	-3	33142.2	33148.1 (0.02%)	33148.0 (0.02%)
	4	42.75	60	-1	32203.3	32216.2 (0.04%)	32215.4 (0.04%)
	6	19.00	40	-1	31879.0	31894.9 (0.05%)	31902.4 (0.07%)
20% Probability of Individual Unit being Nondefective							
Constant	2	171.00	300	-3	73334.3	73337.5 (0.00%)	73337.5 (0.00%)
	4	42.75	150	-1	72602.2	72604.5 (0.00%)	72604.0 (0.00%)
	6	19.00	100	-1	72357.5	72362.8 (0.01%)	72361.9 (0.01%)
Sinusoidal	2	171.00	300	-3	75688.9	75694.1 (0.01%)	75694.3 (0.01%)
	4	42.75	150	-1	74982.5	74992.0 (0.01%)	74991.8 (0.01%)
	6	19.00	100	-1	74740.5	74753.2 (0.02%)	74754.1 (0.02%)

†Values are rounded up to nearest integer.

Table 4. Expected 12 Month Cost of Lot-Sizing Policies (and Proximity of Optimality of Heuristics) with a Mean Demand Rate of 25 Units per Month

Demand	Expected No. of Setups	K	EOQ†	i*†	DP	HEUR 1	HEUR 2
80% Probability of Individual Unit being Nondefective							
Constant	2	427.500	188	-7	41653.8	41669.9 (0.04%)	41656.3 (0.01%)
	4	106.875	94	-3	39664.3	39708.9 (0.11%)	39678.1 (0.03%)
	6	47.500	63	-2	38989.5	39042.9 (0.14%)	39009.4 (0.05%)
Sinusoidal	2	427.500	188	-7	41713.2	41781.2 (0.16%)	41777.3 (0.15%)
	4	106.875	94	-3	39871.3	39962.7 (0.23%)	39947.0 (0.19%)
	6	47.500	63	-2	39236.1	39322.8 (0.22%)	39373.9 (0.35%)
50% Probability of Individual Unit being Nondefective							
Constant	2	427.500	300	-7	81133.4	81141.2 (0.01%)	81137.3 (0.00%)
	4	106.875	150	-3	78765.8	78796.2 (0.04%)	78779.6 (0.02%)
	6	47.500	100	-2	77958.0	78004.9 (0.06%)	77978.2 (0.03%)
Sinusoidal	2	427.500	300	-7	82693.4	82737.9 (0.05%)	82734.5 (0.05%)
	4	106.875	150	-3	80380.2	80445.3 (0.08%)	80430.8 (0.06%)
	6	47.500	100	-2	79576.8	79640.8 (0.08%)	79671.2 (0.12%)
20% Probability of Individual Unit being Nondefective							
Constant	2	427.500	600††	-7	183421.4	183423.5 (0.00%)	183430.6 (0.01%)
	4	106.875	375	-3	181450.9	181469.7 (0.01%)	181466.5 (0.01%)
	6	47.500	250	-2	180824.2	180858.1 (0.02%)	180851.4 (0.02%)
Sinusoidal	2	427.500	600††	-7	189348.2	189363.3 (0.01%)	189374.2 (0.01%)
	4	106.875	375	-3	187329.3	187379.1 (0.03%)	187373.1 (0.02%)
	6	47.500	250	-2	186725.1	186774.7 (0.03%)	186775.9 (0.03%)

†Values are rounded up to nearest integer.

††EOQ set to maximum production quantity.

Within each of Tables 3 and 4 we observe that as the production yield (as measured by the probability of an individual unit being nondefective) decreases, there is a substantial increase in the (optimal) expected twelve month cost. In studying the behavior of the heuristics in this regard, we note that the quality of the heuristic solution increases as the production yield goes down. This suggests that these heuristics have the potential to be extremely effective when employed in low-yield production settings. We also observe that the heuristics consistently perform better on those problems exhibiting a constant demand pattern. This difference in performance diminishes, however, as the production yield decreases. Finally, comparing Tables 3 and 4, we observe that there is very little decrease in the performance of the heuristics as the mean rate of demand increases from 10 to 25 units per period.

6. Conclusions

We have presented exact and approximation algorithms for the variable-yield lot-sizing problem. The procedures were tested and compared on problems having various demand patterns and cost structures. In each case, the production yield was assumed to follow a binomial distribution with an (expected) 80%, 50%, or 20% yield. The outcome of the computational experiments showed that two heuristic procedures based on the (s_t,Q) and the (s_t,S) decision rules consistently generated lot-sizing policies which were within 0.4% of optimality. Moreover, the quality of the heuristic solution improved as the production yield decreased.

While both heuristics performed very well, the hybrid (s,S) decision rule of the HEUR 2 procedure provided the best overall approximate solutions. This procedure provided solutions that were within 0.05% of optimality on average, and never worse than 0.35% of optimality.

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